The Partitioned Quantum Universe: 
Entanglement and 
the Emergence of Functionality

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Abstract

Given that the world as we perceive it appears to be predominantly classical, how can we stabilize quantum effects? Given the fundamental description of our world is quantum mechanical, how do classical phenomena emerge? Answers can be found from the analysis of the scaling properties of modular quantum systems with respect to a given level of description. It is argued that, depending on design, such partitioned quantum systems may support various functions. Despite their local appearance these functions are emergent properties of the system as a whole. With respect to the separation of subject and object such functions of interest are control, simulation, and observation. They are interpreted in close analogy with more basic physical behavior.

1. Introduction

Today we believe that the fundamental description of our world should be based on quantum mechanics. Recent research indicates that this might even be taken literally: The “cosmological horizon remains the same finite size forever if the universe happens to be accelerating forever. One of the implications is that the world as we see it is and will remain a closed quantum mechanical system with only a finite number of degrees of freedom forever” (Yi 2003).

All pertinent models of reality are composite, if for no other reason than to allow for a separation of subject and object. To be consistent with the above statement, we have to accept that also the subject (the human participant) should be governed by quantum rules. While the full meaning of this statement is fairly unclear yet, we intend to model the subject via particular functions which, in turn, are supposed to have some appropriate material basis. This is a variant of Landauer’s famous dictum “information is physical” (Landauer 1991), now translated into “functions are physical”. We will take the liberty to replace the actual (unknown) partitioned quantum system, i.e. its decomposition into subsystems, by
rather primitive schemes reminiscent of current technical devices whenever appropriate.

The functions under consideration here are control, simulation, and observation. They are all interdependent and supposed to be embodied in the actual physical system; the functions are thus expected to result from partitioned quantum systems rather than being implemented symbolically. Observation is taken to mean gathering of information about the outside world, including measurement and storage. Simulation is understood as a tool to understand, to interpret, to find a model of what is observed. (In a somewhat technical sense I can say that I understand a behavior to the extent that I can simulate it (Nicolis 1986).) Control allows to influence part of the environment in some desired direction, motivated by understanding and conditioned by (system) design. Not accidentally, all these functions are strongly related to the concepts of information science.

Within physics information plays a number of different roles, some of which are rather technical, others more fundamental. Fundamental questions relate to the meaning of information as a relational (contextual) property and to the notions of completeness and incompleteness. “With Lucretius, I find disorder in an initial state of the universe repugnant. Furthermore, a pure state remains pure. Entropy is somehow to be developed without fundamental entropy... Indeed my favorite key to understanding quantum mechanics is that a subsystem cannot be isolated by tracing from an enveloping pure state without generating impurity” (Lubkin 1978).

The origin of incomplete knowledge has bothered researchers over many centuries. In classical physics limited information can always be traced back to subjective ignorance. Quantum mechanics offers a new and intriguing avenue to combining complete order of the whole with objective indeterminism woven into the reality of any of its parts.

Ironically, though, just those statistical features give also rise to classical phenomena on the macroscopic scale (cf. Sect. 3.1 and Gemmer et al. 2001). As a consequence we face a confusing tension: Should we expect a strange quantum world, up to now being kept beneath some classical surface, to be unleashed in the future? And will this lead to new technologies (like quantum computing) and/or new types of theories and models for understanding our position in this universe? Or will quantum mechanics remain somewhat “hidden” and accessible mainly in an indirect way?

Meanwhile it has become fashionable to use quantum computing (Nielsen and Chuang 2000) as a metaphor for unheard-of non-classical power that could possibly underly psychological phenomena, human perceptions and brain functions, free will or even consciousness, to name but the most spectacular directions. However, before one dares to venture out to such new frontiers, it should be wise to critically review the present status of
research in the field of quantum information. This research is plagued by severe scaling problems. Are these of temporary technical origin only or indicative of fundamental limitations?

What do we mean by “scaling”? The difference between power laws and exponential laws plays a significant role in many branches of science. Power laws have no internal scale and are said to describe scale-invariant (scale-free, self-similar; Schroeder 1991) properties. Exponential laws necessarily contain a scale (in the exponent) and thus count as “unscalable”. Many fundamental laws of physics are scale-invariant (e.g. the Coulomb force); material properties referring to a particular length-scale (resolution) are not, as the hierarchical structure imposes various fixed length scales. In any case scaling refers to a given functional dependence – one and the same system may scale differently depending on the type of questions asked.

Our interest here is focused on efforts (efficiencies) to implement the functions mentioned above, i.e. to control, to simulate, to observe. Quantitative measures of those functions will have to be based on “counting numbers”, i.e. counting parameters, steps, procedures etc.

2. State Space

2.1 Reference Frames

States, whether classical or quantum, have to be specified by parameters (data, i.e. information in the most basic sense). The meaning of these parameters, though, is defined with respect to a given reference frame only. For a classical point particle there are 6 real parameters (3 with respect to a coordinate frame for position, 3 for the momentum). For an \( N \)-particle system there are consequently \( d_c = 6N \) independent parameters, which define a point (state vector) in the so-called state space \( \Gamma \). This abstract space has \( d_c \) dimensions.

In the quantum domain it is more convenient to consider an object with a finite number, \( n \), of orthogonal (pure) states. Such basis states are said to constitute distinguishable alternatives in which the object can be found. The dimension of its so-called Hilbert space is thus \( n \). Any other pure state can be represented as a superposition of those orthogonal basis states.

However, a state does not need to be pure (i.e. specified without lack of knowledge, with zero entropy \( S \)) in general. Non-pure states are defined by a so-called density operator \( \hat{\rho} \). We will return to the possible origin of the associated uncertainty described by a positive entropy \( S \) in Section 5.1.

Operators can be defined by their action on a complete set of basis states. At first sight it may, thus, come as a surprise that a state itself
should be an operator. In fact, this is an example of an implicit operational definition: Any operator can be specified by its action on those basis states. In quantum mechanics both general states and observables can be represented by operators. An additional rule then allows us to calculate the expectation value of an observable in a state (something relating to our experience): this rule is simply a generalization of the scalar product, the trace relation, between state and observable.

For a finite Hilbert space of dimension $n$ there are $p_1 = n^2$ different (i.e. orthogonal) operators forming a complete set out of which we can compose any other operator. For the operator $\hat{\rho}$ the normalization represents one additional constraint, so that there are $d_q = p_1 - 1$ independent real parameters. They tell us to what extent any of these elementary operators is present within the density operator considered.

Now, those $d_q$ parameters can be taken to specify a point in the so-called Liouville space of $d_q$ dimensions. In contrast to the $\Gamma$-space, the Liouville space has a finite volume, because the eigenvalues of all operators are bounded here. For a spin with $n = 2$ the $d_q = 3$-dimensional Liouville space is a sphere, the so-called Bloch sphere. The pure states have maximal length and form its surface.

2.2 Partitions

The decomposition of a quantum universe into subsystems is unique only in terms of elementary quantum objects. (Most of those will be indistinguishable, so that further restrictions due to permutation symmetry apply). Take, for example, a toy universe consisting of $N$ identical objects. Its state can uniquely be characterized by a set of $d_q = p_N - 1$ independent parameters, which tell us, to what extent any of the $p_N = n^{2N}$ so-called product operators is present within the respective density operator (Mahler and Weberuss 1998).

Product operators are a bit like a chord on a piano: We can either hit or leave untouched any key at any time (two alternatives). In case of the “quantum keyboard” for spin networks ($n = 2$), however, we have $p_1 = 4$ alternatives including the “do not touch” option. This latter operation is the so-called 1-operator, taken here as one member of the complete, orthogonal set. In any such “chord” between 0 and $N$ spins are actually worked on. Given there are $N$ keys on the keyboard, this gives $p_N = 2^{2N}$ different possibilities to play. Of course, classically we could only strike one chord at a time, but within a quantum state all these chords can be present in a kind of a superposition of different actions.

In the classical case the local data on all individual point particles, $d_c$ degrees of freedom, completely define the state of the system as a whole. In the quantum case the local data, $3N$ parameters to specify each individual spin, rapidly become insufficient. In fact, the ratio $\gamma = d_q/3N$
grows exponentially with $N$: $\gamma = 7$ for $N = 3$, but we already have $\gamma = 3 \cdot 10^4$ for $N = 10$. What is this overwhelmingly large amount of surplus data needed for?

The exponential blow-up in the number of independent parameters is a typical quantum phenomenon, resulting from the non-commutativity of the respective operators. This is much closer to the heart of quantum science than the famous uncertainty relations. A state without any entanglement (i.e. quantum correlations) would always be characterized by $\gamma = 1$; pairwise entanglement would lead to an additional contribution in the parameter number of order $O(N^2)$ and thus to a $\gamma \approx N$; higher but still finite order entanglement would add further terms polynomial in $N$. If we admit unrestricted entanglement, we need exponentially many parameters in order to specify the state, as given by $d_q$.

We note in passing that entanglement is – just like any one-particle superposition – not an absolute property but depends on the chosen representation. What appears to be a superposition with respect to one set of states (i.e. one reference frame) may look like a single pure state with respect to another. While the meaning of parameters is always relational, the existence or non-existence of entanglement makes a qualitative difference.

Any fundamental partitioning of a system into subsystems is usually far from a pertinent level of description. At higher levels, though, e.g. considering groups of elementary objects, many different partitions may result. Each of them forms a kind of reference frame, and the corresponding parameters can be transformed from one representation to the other. Let $\alpha$ be a factor of $N$; then we could re-arrange, e.g., the objects into $N/\alpha$ effective subsystems of Hilbert-space dimension $n^\alpha$ each. Of course, the number of parameters, $p_N - 1$, would remain the same.

Eventually we could consider a bi-partite system as envisioned for the separation of subject and object. Note that the respective single “cut” is not the so-called Heisenberg cut: Both parts are assumed to be fully quantum mechanical and the description of the whole is assumed to be complete.

However, the equivalence between different decompositions is lifted once we consider the partitioning as “real” rather than a mere mathematical procedure. A real partition is understood to mean that all correlations crossing the borderlines between the selected parts are taken to be unobservable. Different partitions are thus characterized by different pieces of missing information. They have become incompatible – defining, in a sense, different realities. The selection of a “real” partition will thus have to be of prime concern (cf. Sect. 6).
3. Control

Given that the world as we perceive it appears to be predominantly classical, how can we stabilize quantum effects? Given the fundamental description of our world is quantum mechanical, how do classical phenomena emerge? Answers to these questions can be found from the analysis of the scaling properties of quantum systems that are partitioned with respect to a given level of description.

As regards the separation of subject and object, a most complex kind of partition, we tentatively assume that the functions pertaining to it will, in principle, be similar to emergent functions of other embedded quantum systems including technical devices. We may thus not only learn from nature (i.e. from the environment as perceived) how to solve engineering tasks, but also learn from engineering solutions how to arrive at a better understanding of nature.

3.1 Classical Regime

Limited knowledge about a system and limited access to the environment from which some subsystem may actively be controlled can, in principle, not be avoided. One may try to take advantage of the resulting randomness. However, a much more realistic strategy is to minimize the effect of those limitations by making the system dynamics (almost) independent of the parameter details. This is the thermodynamic level of description. This mode of control (close to local equilibrium) is extremely stable and robust; it is classical in the sense that the pertinent thermodynamic variables are no quantum operators and are sharp in the thermodynamic limit.

Large quantum systems have been the subject of theoretical physics for decades (Thirring 1983). Decoherence as an inevitable effect of quantum systems interacting with a quantum environment has been identified as the source of thermodynamic behavior and of the second law (Gemmer et al. 2001, Gemmer and Mahler 2003). The maximum entropy principle thus follows from entanglement between the system considered and its environment. This is similar to the discussion in Section 5.1, except that the accessible Hilbert space is now constrained by additional invariants like energy distributions etc. Entanglement (a property of the total system) thus gives rise to local classicality. Macroscopic variables are recognized as those which become dynamically stabilized. These deterministic variables play a central role in thermodynamics as well as in traditional physical experiments. But note that the temperature of a system, e.g., is not an internal property of that system; it rather follows from the spectral properties of its embedding (Gemmer et al. 2004).

Thermodynamic behavior abounds and, not surprisingly, there are only very few conditions for this to happen. One of them is the weak coupling
requirement. Irrespective of details, with increasing length scale the residual coupling between two subsystems decreases simply because boundary effects go down with respect to volume effects (if interactions are short-ranged). A partitioning of system and environment on macroscopic length scales will thus lead to weak effective coupling. The extremely limited (subjective) knowledge about initial states and Hamiltonian models then turns out to be fairly irrelevant for predicting the equilibrium state (characterized by objective uncertainty with respect to microstates). However, after the system has reached its equilibrium, there is no dynamics anymore.

Stationary non-equilibrium situations occur within so-called transport scenarios. A system $g$ then has to respond to an environment $c$ partitioned into (at least) two separate uncoupled parts each of which, if present alone, would induce a different temperature on the system $g$. This embedding requires a further partitioning of the system $g$. Local thermodynamical equilibrium still applies, provided the (mesoscopic) length scale for this partitioning is large enough. The interplay between the “objective” physical structure and the “appropriate” partitioning is particularly transparent in this scenario.

3.2 Quasi-Classical Regime

It has been recognized recently that thermodynamic behavior should emerge already within surprisingly small systems and environments (“small worlds”; Jensen and Shankar 1985, Borowski et al. 2003). If one accepts increased fluctuations (due to finite size effects), a universal quasi-deterministic description can be expected to hold (after relaxation) down to any 1- or 2-spin subsystem within a closed network with as few as 8 or 10 spins! Note that this (universal) local behavior would coexist with a global coherent dynamics, which, however, is assumed to be undetectable here. For weak coupling such an ultimate (nanoscopic) length scale should even apply to non-equilibrium scenarios (Michel et al. 2003). A larger coupling can be compensated by an increased reference length scale on which the effective coupling would become small again (Hartmann et al. 2004). These findings indicate that some quasi-classical mode of operation (i.e. the $\gamma \approx 1$-state subspace) may indeed be available far down into the nano-world.

There are several proposals for taming nanoscopic systems. However, even if quantum features proper are not yet the target with respect to information processing (i.e. the control space is still constrained to $\gamma = 1$), scalability already tends to become a major problem. While the operation of individual elements can usually be demonstrated, the design of the most appropriate type of architecture, let alone the actual operation of a whole array, becomes a daunting task.
Single electron devices (Wang 2002) try to exploit the Coulomb blockade effect. Single electron transistors and few combinations of those have been demonstrated experimentally. The pertinent architecture is expected to be the same as for conventional electronic devices. Low temperature operation conditions and extreme fabrication problems have severely limited further progress so far.

Molecular electronics (Mahler et al. 1996, Wang 2002) has been under investigation for decades. Some elementary devices have been demonstrated, but the construction of useful networks remains a severe challenge. A scalable solution is not in sight. Proposals to enhance reliability via ensemble averaging may indeed help but lead back to mesoscopic rather than molecular designs proper.

Another approach is based on quantum cellular automata (QCA) using semiconductor quantum dot technology (Lent et al. 1993). (The name QCA refers only to quantum size effects; no quantum coherence whatsoever is being exploited.) Since its proposal in the early 1990s, it has attracted much attention because of its promise of low power consumption and scalability. The output of a computation is obtained by enforcing boundary conditions on the edge of a two-dimensional array of bistable cells with nearest neighbor interaction. Bistability is obtained with respect to different electronic charge distributions within the nanostructured cells. The result of the computation would be read out after relaxation as part of the ground state at the opposite edge of the array.

After ten years of research and much initial enthusiasm it is now clear that a viable large-scale QCA is beyond the reach of (current) technology (Girlanda and Macucci 2002). The reason is that the electrostatic implementation is associated with extreme sensitivity to fabrication details of the individual cells, stray charges and unwanted couplings. Note that this disappointing result is in strict conflict with previous naive claims of scalability. After all, scalability is not a theoretical (i.e. model-oriented) but a practical concept.

3.3 Quantum Regime

As physical structures are not scale-invariant, the shrinking of pertinent internal length scales cannot continue indefinitely. With respect to device physics, “Moore’s law” (i.e. doubling of memory at fixed cost within every 18 months; Moore 1965) will eventually be abandoned.

As mentioned, weak coupling is a prerequisite for thermodynamic behavior (internally as well as with respect to the system in its environment). If this does not hold on the length scale studied, the conventional mode of description (i.e. a local variant of thermodynamics) is no longer available.

Even in this case, the coherent dynamics of the total (closed) system is still guaranteed. Unfortunately, this fact is of little practical concern,
as we can hope to control, at most, a tiny (modular) subsystem. The time scale for coherent subsystem-dynamics within the accessible control space will thus necessarily be finite. Furthermore, on small length scales the hierarchical ordering of coupling parameters can no longer be taken for granted. Next-nearest-neighbor in addition to nearest-neighbor couplings (or opening up of tunneling paths which should be blocked etc.) have to be taken into account in parallel. As a consequence the local modules lose their well-defined identity and the appropriate dynamical description of the network is no longer well-defined. And in contrast to the macroscopic regime, any lack of information about the system parameters will be reflected in a loss of dynamical control (Skinner et al. 2003).

Special design (e.g. structures to enhance the parameter hierarchy or to support special symmetries for decoherence-free subspaces (Nielsen and Chuang 2000) and stroboscopic “refreshing cycles” (error correction; Steane 2001) might increase the coherence time or, at least, prevent the system from approaching equilibrium. An extended control space with $\gamma$ significantly above 1 could become accessible.

Note, however, that the present idea of quantum computing would require to force the quantum system into a prescribed sequence of time-discrete quantum gate steps. We thus should be able to efficiently suppress the “natural” eigen-dynamics of the system, e.g. to keep any state of the system frozen at will – an almost impossible feat indeed.

The conventional implementation approach has been “bottom up”, i.e. one tries to manufacture quantum networks step by step with a gradually increasing number of modules, $N = 1, 2, 3 \cdots$ (cf. the quantum computation roadmap at qist.lanl.gov). Pertinent examples are as diverse as cold ions in an electromagnetical trap under optical control and nuclear spins fixed on a molecule and subject to nuclear magnetic resonance (NMR). As the maximum value of $\gamma$ is still small then, complete quantum control is feasible: Everything can be made to work on small enough state spaces, a quite general experience with respect to any complex system. This strategy cannot go on for long, though. The insight (from a fundamental research point of view) gained by extending a 7-spin network to a 8-spin version is marginal, while the increase in control complexity is not.

In an alternative “top-down” approach one starts, instead, from large networks. Examples are so-called optical lattices loaded with millions of atoms (Hamann et al. 1998), solid arrays of millions of semiconductor quantum dots, or millions of defects in a matrix (“spectral hole burning quantum computer”; cf. qist.lanl.gov). These systems are scalable with respect to low-level control (in the quasi-classical regime), but very unlikely so, if the control space was to include the full range of $\gamma$. (Note that this full range is the space of microstates in thermostatistical terminology!)

One may, then, ask whether some restricted quantum mechanical state
space (with, e.g., $\gamma \leq 10$) could be of advantage. However, finite entanglement (polynomial parameter numbers) cannot give rise to an exponential efficiency gain. To what extent “a little bit of entanglement” may support specific algorithms (and not only imply increased instability) is not known yet. Given the difficulties to find interesting applications for the full quantum regime, it is probably even harder to find a candidate for using such restricted quantum features. The (algorithmic) complexity class would remain the same anyway.

4. Simulation Space

4.1 Complexity

Simulation as a tool to generate understanding can be formalized: Originally and according to Turing’s initial thinking, a computer was conceived as nothing else than the substitute of a human being carrying out a sequence of formal procedures (Hodges 1995). This sequence could, then, represent a kind of internal model for some observed structure within the environment.

Nowadays it is more convenient to think of replacing such a human being by an appropriately designed computing machine, thus supposing that mental processes require a physical implementation. This is a symbolic (software-oriented) use of a physical system. In order to be useful such a machine requires modularity. For this reason, universal digital computers are based on a finite set of elementary gates, the sequential combination of which generates the desired algorithm, physically equivalent to a transformation from an initial state (input) to a final state (result). This can be done in more or less efficient ways. A convenient measure of efficiency consists in counting the number of steps (proportional to time) needed for a specific algorithm. The corresponding algorithmic complexity tells us how this number of steps $s$ depends on the size of the input $i$: For an “easy” (executable) problem this is polynomial or just a power law, $s \approx i^\alpha$ (class $P$), for a “hard” problem it may be exponential or worse (e.g. class $NP$, “non-deterministic polynomial”; cf. Nielsen and Chuang 2000).

Is this complexity a property of the algorithm only? One might expect that it depends also on the type of gates, which, in turn, depend on the type of physical implementation. Indeed, one could imagine a gate that solves a special purpose problem (even a hard decision problem, say) in a few steps, whereas other gates would require very many.

Such kinds of “miracles” would indeed become available if some delicate extensions of physical rules could be assumed. A classical computer with real number representation of infinite precision (Schönhage 1979) or a computer based on a non-linear version of quantum mechanics (Abrams and Lloyd 1998) would allow to solve $NP$-complete problems efficiently.
A quantum network would make a hard satisfiability problem easy, if optically induced many-particle transitions were available without limits (Zak 1999). The factorization of any number would become a two-step process, if multi-photon states could be prepared efficiently (Alicki 2000).

Currently the best known and extensively discussed example is Shore's factorization algorithm (Nielsen and Chuang 2000, Lavor et al. 2003). Its exponential increase of efficiency over any known classical procedure (cf. Sect. 4.2) can be traced back to the use of the so-called quantum Fourier transform. It would work if large-scale quantum computation could be implemented in an efficient way.

Unfortunately, the conditions supporting any such “miracles” can typically not be fulfilled. In most cases this is not because of an explicit violation of a physical law but because of unsurmountable control complexity. We may call a situation “physically hard” (or unscalable), if control and implementation efforts grow exponentially with system size. But unless we are told by experience, there is often no simple way to decide whether a procedure is hard or not: If we throw a stone into the center of a lake, we easily generate beautiful circular surface waves. But it is very difficult (hard?) to generate the reverse pattern by preparing appropriate initial conditions at the shoreline – a procedure that is not forbidden by fundamental physical laws.

It is now generally accepted that for any standard classical computer (not requiring those physically hard procedures) the classes of “easy” and “hard” algorithms, respectively, are the same. As a consequence these computers can simulate each other with at most polynomial overhead. This somewhat surprising equivalence postulate (“Church-Turing thesis”; Nielsen and Chuang 2000) has made information theory a field of research on its own and fairly independent of physics.

Whether or not this equivalence will eventually be violated at the quantum level is unknown yet. The full control of large-scale quantum networks is beyond current technology. Simulation – based on physics – repeats the whole quest for functionality within a partitioned quantum universe on a smaller scale. To what extent can the respective computer operation (or mental process) be made a stable function?

4.2 Hamilton Models and Trajectories

The quantum systems under consideration are defined as so-called Hamilton models. These models are operators characterized in the same Liouville space as the corresponding general states and thus require, in principle, as many independent parameters. Their number can be reduced to a polynomial in \( N \) (for a modular system composed of \( N \) subsystems) by taking into account the topological structure, i.e. by incorporating the fact that interactions are typically pairwise only and depend on distance.
As a result parameters come in a hierarchy ranging between strong and weak coupling.

Spatial homogeneity would further reduce the pertinent number. However, homogeneity is often violated. There are fluctuations in local parameters if the modules are created in a basically random process (like semiconductor quantum dots) rather than being elementary units (like specific molecules). Fluctuations in distance between the modules will lead to fluctuations in the pertinent inter-subsystem coupling.

The Hamiltonian does not only specify the system but also generates its deterministic dynamics (via the Schrödinger equation). The corresponding quantum trajectory is uniquely given once the initial state is known.

A quantum trajectory, starting from an unentangled state and being generated by an Hamiltonian with polynomially many parameters, may well traverse complex state spaces with large $\gamma$, apparently without imposing “hard” control problems (cf. Sect. 4.1). This possibility, typically introduced via explicitly time-dependent Hamiltonians is believed to be the major source for the potential efficiency gain encountered in (pure-state) quantum algorithms. Indeed, the exponential gain expected for Shore’s factorization scheme has been shown to be related to the fact that the quantum trajectory is forced to visit states of unlimited entanglement between preparation and read-out steps (Josza and Linden 2002). Both state classes, $\gamma \approx 1$ and $\gamma \approx \frac{d_4}{N}$, represent pure states ($S = 0$). Their joint availability may be regarded as an essential feature of quantum networks supporting quantum computation. Entanglement would, thus, act as a precious resource for this function.

4.3 Embodiment

Artificial intelligence has been a prime target of advanced simulation (Dreyfus and Dreyfus 1988). One group of researchers tried to model the mind’s symbolic representation of the world by means of a computer. Others intended to design a physical device (a neural network) generating the desired abilities. Both lines of thought are based on the idea that function is defined locally. Wittgenstein (1975) argued that the analysis of everyday situations in terms of facts and rules is itself only meaningful in some context and for some purpose. Functionality as discussed in the context of partitioned quantum networks strongly supports this view.

In contrast to the universal equivalence principle, quantum information processing depends, at least in principle, on the underlying implementation. This fact may be taken as a sign of a new trend paving the way back to complex functions emerging at the interface between real physical systems.

As a most basic example one may refer to quantum key distribution (better, though erroneously, known as quantum cryptography; Nielsen
and Chuang 2000). Here the local spin is realized as the polarization state of individual photons. Photon pairs entangled in this polarization subspace are then used to generate strictly correlated information at the location of two measurement detectors $A$ and $B$. While illegal eavesdropping can never be ruled out, its presence can be tested by $A$ and $B$, and thus avoided by repeating the trial. This non-classical possibility rests upon the sensitivity of the entangled pair states with respect to any local measurement, in particular that of the eavesdropper.

Note that the whole process could easily be simulated on a classical computer. This, however, would entirely be irrelevant as quantum key distribution is not a symbolic process but, rather, embodied within a particular physical context. The environment is an essential part of the desired function. Other examples abound: one cannot drink simulated water, a simulated superconductor is not superconducting (with respect to its natural environment).

As a further class of scenarios one may contemplate the use of external coherent control for the design of novel properties of a material subsystem. These properties would then be the output, and in that sense the quantum features remain embodied in the material carrier. As a simple example one can refer to the ultra-slow group velocity of light as generated recently within a coherently driven ensemble of three-level systems (Hau 1999). Such externally driven materials have previously been proposed as “smart matter” (Hogg and Chase 1996). Quantum effects should become important insofar as non-equilibrium effects are stabilized on various scales simultaneously.

Eventually, embodiment can be discussed within a much broader context. After a long series of failures essentially based on the abstract computer paradigm, present-day artificial intelligence and robotics are now reconsidering “embodied intelligence” (Cruse et al. 2002). It is all too obvious that the walking behavior of insects, say, cannot be controlled by a central computer: the neural networks would simply be unable to provide the immense computational power needed. First steps have been undertaken to extend robotics to the quantum domain (Benioff 2002), where not only the central processing unit, but also the environment is made quantum. This example differs from the open quantum computer system discussed before (Sect. 4.2), since the interface is physical rather than purely informational. For the usual concept of a quantum computer the influence of the environment is detrimental due to decoherence (see Sect. 5.1), but has no functional significance.
5. Observation
5.1 The Decoherence Problem

Closed quantum systems are, at most, a convenient fiction. Any realistic quantum system (except the universe as a whole), whether itself modular or not, necessarily interacts with a quantum environment. In general, interactions lead to entanglement and well-defined correlations (sharp global properties within a pure state of the whole), implying that local properties tend to become uncertain. The subsystem of interest can only be described by a local density operator $\hat{\rho}$ with finite entropy $S$. This is reminiscent of the well-known incompatibility between the position of a particle and its momentum, as expressed in the Heisenberg uncertainty relation. The appearance of local uncertainty is known as the notorious decoherence problem (Zeh 1996).

Without explicit reference to the underlying Hamiltonian (cf. Sect. 4.2), the decoherence problem can be given a very general but nevertheless simple and intuitive form. We only have to specify the number of different states of the system ($gas$ $g$, Hilbert space dimension $n_g$) and of its environment (container $c$, dimension $n_c$), and assume the total state (defined in the Hilbert space with dimension $n_{tot} = n_g \cdot n_c$) to be pure.

Based on this extremely limited (subjective) knowledge we assume the actual pure state $|\Psi\rangle$ to be a member of an ensemble characterized by some probability density $w$. This probability density is defined on the set of parameters specifying the state. While the type of parameters depends on the chosen representation (just like the coordinates of a vector depend on the reference frame; Sect. 2.1), the unitary invariant density that we use looks the same in any representation, i.e. it is invariant and thus "unbiased" (Lubkin 1978, Skyora 1974).

With the help of the Hilbert space-density $w$ we can calculate the probability of any functional associated with the state function, in particular of properties associated with the individual subsystems. An interesting functional of this type is the von Neumann entropy, $S_g(|\Psi\rangle)$, for subsystem $g$. In Fig. 1 we see that for $n_g = 2$ the ensemble average of this entropy, $\langle\langle S_g \rangle\rangle$, rapidly approaches its maximum value as the dimension $n_c$ of the environment increases (Mahler et al. 2003). This is a consequence of the entanglement between system and environment – highly entangled states are thus typical and by no means the exception. This means that without any constraints local system properties are most likely completely uncertain. This is an interesting result: While we do not know the actual pure state of the whole system, this (subjective) ignorance is irrelevant. Almost all states have the same local properties in terms of an objective uncertainty described by a finite local entropy! The appearance of such dominant regions is generic for any bipartite system and eventually gives rise to (local) thermodynamic behavior (cf. Sect. 3.1).
5.2 The Measurement Problem

Measurement requires a (logical) correlation between the measurement apparatus and the measured object. As such it appears to be just another example of a system-environment interaction, as discussed in Sect. 5.1. However, the resulting entanglement would imply indeterminate local properties, including indeterminate pointer readings of the apparatus. In any real experiment, however, the pointer is found to have “collapsed” into a specific position. Repeated measurement outcomes occur with the probabilities as predicted from the wavefunction $|\Psi\rangle$ before the collapse. This statistical feature is a basic ingredient of the standard interpretation of quantum mechanics, yet it remains dynamically unexplained and constitutes the so-called quantum measurement problem. It has led to speculations that a solution of this problem might eventually require a conscious observer (cf. Bierman 2003).

Let us, instead, try to address the problem from within physics, in particular from the functional perspective as introduced in Sect. 1. We have to admit then that entanglement appears to have many faces: In the gas-container model of Sect. 5.1 it gives rise to unsharp local properties and thermodynamics, in accordance with everyday experience. In the object-apparatus model entanglement (though required to generate the desired measurement logic) is eventually destroyed as new information is locally generated. We learn something which has been undefined before, not just unknown to us. But what could be the material basis of this functional difference? We propose to consider any satisfactory answer to this question a solution to the measurement problem.

For quantum measurement scenarios, non-equilibrium is essential, while the clicks of a photo-detector, say, eventually re-establish equilibrium. But
the measurement dynamics is not just relaxation. The probability to find
a single photon behind a beam-splitter is 50% for either path. Both paths
are degenerate, there is no preference whatsoever. However, only one of
the two detectors will eventually click: symmetry breaking and informa-
tion gain play a prominent role.

In order to become a fact, the measurement outcome has to be regis-
tered. The information has to be stored in a memory. The stability of this
memory requires the emergence of an approximate “superselection rule”
(Ballentine 1998), according to which spontaneous transitions between
the storage states is suppressed for all practical purposes. The implementa-
tion of this measurement function implies a sort of self-reference. Part
of the total state space of the system has to be reserved for the storage of
another part of the state, referring, e.g., to some specific previous instant
of time.

Obviously, such a task does not scale up. The system cannot evolve
and at the same time keep a record of its entire history. For this reason we
cannot claim that in the gas-container-model the container continuously
monitors the microscopic state of the gas. Quite to the contrary, even if
some information about the gas would become available, thus changing the
momentary state of the gas, this (subjective) gain in information would
have no relevance whatsoever for the objective entropy characterizing the
equilibrium state: the momentary information gain is lost in relaxation.

Partitioned systems with mutual interactions should thus admit quite
different functional behavior. While the Schrödinger dynamics suffices to
account for thermodynamics within a subsystem, quantum measurement
can only be followed up to the point where the local state can safely be
interpreted to consist of a set of stable alternatives with given (classical)
probabilities. The “final jump” is not contained in the theory. This funda-
mental indeterminacy can be modeled by stochastic trajectories, gen-
erated by throwing dice, a basic act of creativity. “The probabilities as-
associated with measurement develop because the observer must implicitly
trace himself away from the observed system” (Lubkin 1978).

5.3 Perception

Physicists traditionally refuse to include observers within their model-
ing except in an abstract way. For instance, reference frames are defined
within relativity theory, observables are introduced within quantum me-
chanics. A more explicit quantum model of an observer and its interface
with the observed system has rarely been considered so far (cf. Primas
1990). The observer as a genuine part of a quantum system as a whole
may lead to problems of self-reference similar to those implied by mea-
surement and the need to register.

We may expect perception to present a rather simple example of the
measurement function, basically as discussed in the preceding section.
While our senses involve rather diverse types of detection devices, from those working close to the quantum limit, like our eyes (Ricke and Baylor 1998), to those which are rather close to equilibrium physics, like the detection of warm and cold, these different schemes should not impose difficulties in principle. Clearly, there are physical limits to the temporal and spatial resolution accessible to our sensory systems.

However, beyond these sensory limits the stream of states must be interpreted, which is impossible without a partition. In fact, the meaning of parameters as discussed in Sect. 2.1 is based on such a partition. But how come the partitioning? It must be a consequence of “design”, built into the measurement scenario as a whole. We are led to assume that our perception, despite its internal appearance, nevertheless remains part of the total system (cf. Sect. 6). A different partitioning could lead to extremely strange modes of perception – which indeed appear to be possible (Sacks 1985).

Perception in terms of a simple data acquisition scheme seems to be rather classical. However, seen as an embedded function (like that of generating equilibrium), a quantum description should become mandatory. Such a description may allow us to address a subtle aspect of human perception: the fact that the observer must be aware of the status of his observation. For instance, he must be able to ascribe any modified properties of his retina, say, as related to the outside world and not just to his internal dynamical evolution. Only in this way does that structural change “within” become a piece of information “about”. Classically, correlation requires an outside perspective. In the quantum domain correlations are also locally present in the sense that they make local properties indeterminate. In such a non-local model, the notorious binding problem (i.e. the integration of different sensory stimuli) might appear as the consequence of an ill-posed question.

6. Adaptive Partitioning

Thermo-mechanical systems require both thermal and mechanical contacts. Large-scale limits are well-known to constitute thermodynamical machines. These machines are structures that can create non-equilibrium out of equilibrium by exploiting mechanical (electrical, chemical) degrees of freedom. It is an interesting though still unsolved problem to study their behavior in the full quantum regime. The design of the total system plays a decisive role in this context. The sharp distinction between classical and quantum features will eventually be lost in this case.

The journey into the world conceived as a quantum network has just begun. A web of even more complex behavior may eventually be uncovered without any need to postulate basically new rules. However, we have
to accept that local functions do not necessarily have a local explanation. These extended models do not have to be new devices, ready for application. To understand the transition from non-living to living matter may require a similar approach (Rasmussen et al. 2004). If life can be described as something like a “dressed state”, then neither artificial life in a natural environment nor natural life in an entirely artificial environment are possible. Correspondingly, there is no unique or “true” way to partition an organism into functional subsystems such as organs (Lewontin 2001). And while genes are often characterized as a local program of life, this picture has become untenable (Keller 2000) despite its popularity. This basic view is also supported from a different perspective: Berlin (1996) has observed that one cannot simulate human life of previous epochs, since the open list of implicit knowledge (the context) is not available at the level of the individual actors.

It is unlikely that the level of description for such measurement scenarios is prescribed (cf. Vecchi 2001). While we accept that the world has a particular structure (design), partitions are not given “out there”. The world as we see it has no fixed meaning. Particular interpretations should be the result of an adaptive (self-referential) dynamical process by itself. One may tentatively say that the “optimal” partitioning should reflect the “objective” structure of the universe (cf. Shalizi and Moore 2003), taking into account the hierarchy of strongly and weakly interacting subsystems.

Furthermore, one may speculate that (human) perception continuously tries to establish a partitioning of the world that tends to make this world appear predominantly classical and intelligible. Ironically, modern quantum experiments are at variance with this traditional optimization process, leading to bewildering experiences.

This optimization would have to be part of the act of perception (cf. Sect. 5.3), introduced as an emergent function at the interface between subject and environment. Seen as a dynamical process reaching far into the realm of model generation (simulation), this optimization could continuously be subject to change and even become ambiguous (Blackmore and Troscianko 1989). Large (and therefore rare) fluctuations within the physical carrier of that function may even give rise to phenomena outside the domain of established science.

Such an optimization procedure would have a simple analogy in the descriptive partitioning of our physical environment. “Dressed” subsystems or so-called elementary excitations in solids have reduced mutual effective interactions, rendering them more classical. Theoretically, one could imagine to have diagonalized the Hamiltonian of the entire system of interest (cf. Sect. 4.2), i.e. to have found a reference frame in which all states are non-interacting. However, as we and our measuring instruments are part of the game, such a solution would be meaningless.
7. Summary and Conclusions

“Information is physical” (Landauer 1991) is a somewhat ambiguous statement. It certainly means that information needs a physical carrier. But, contrary to widespread claims, it also means that information is an emergent rather than a fundamental property of a partitioned system, which is able to define a reference frame.

Large-scale quantum control may well turn out to be physically hard. Neither technology nor nature can exploit modes of operation that are inherently unstable. This should also severely limit the significance of direct and locally coherent quantum processes in models of brain and consciousness.

It has been shown that partitioned quantum systems support specific functions. Depending on system design and choice of partitioning these functions emerge locally even though the dynamics of the total system continues to follow the standard Schrödinger dynamics. But despite their local appearance these functions are nevertheless emergent properties of the system as a whole. Non-locality in this sense is thus widespread and not restricted to basic quantum phenomena. Macroscopic non-locality, to be sure, does not mean that I can walk through two different doors at the same time. It rather means that my properties cannot be seen separate from my environment in the most general sense.

Emergent functionality may thus be a more promising concept than to continue the trend towards control on ever smaller length scales. In this way a web of complex behavior may eventually be uncovered. However, this will only happen if we accept that local functions do not necessarily have a local explanation. Eventually this is the dilemma of any scientific endeavour: dividing the indivisible. As examples of such functions we have considered:

- **Thermodynamic behavior**: The thermodynamic state is a state of the system but enforced from and stabilized by its quantum embedding.

- **Quantum key distribution**: The security of the protocol is a property of the entangled photon pairs, but also of its quantum environment, which any eavesdropping has to make use of.

- **Observation**: The taste of a fruit depends on the fruit but also on the observer. Though we have not been able to present a complete observational model, the paradigm of emergent functionality appears to be quite promising in this respect.

- **Control**: The dynamics of the controlled system depends on the system but also on its control environment (e.g. dressed states).
• **Simulation:** Information processing is symbolic but also needs physical implementation and control. With further down-scaling, the physical properties of the simulator start to call for a simulated environment to extract information as desired.

• **Walking motion,** while based on a program of the controlling brain, also sensitively responds to the mechanical properties of the legs in their environment, etc.

• **Adaptive partitioning:** The selection of a “real” partitioning has tentatively been assumed to happen in connection with (human) observation.

Much further work is needed to systematically explore complex and non-local functionality within multi-partite quantum systems. Such a program, though, may easily become self-referential. Unavoidable approximations impose a kind of effective partitioning, the “reality” of which remains undecidable.

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